# GCSE Maths – Algebra

## Solving Quadratic Inequalities

## (Higher Only)

### Worksheet

NOTES

SOLUTIONS

This worksheet will show you how to work out different types of quadratic inequalities questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

## Worked Example

**Solve the quadratic inequality $x^2 - 11 \geq -2x - 3$.**

**Present your answer on a number line.**

**Step 1**: Rearrange the inequality so that the left-hand side of the inequality is in the form $ax^2 + bx + c$.

$$x^2 - 11 \geq -2x - 3$$

*Add $2x$ to both sides of the equation:*

$$x^2 + 2x - 11 \geq -3$$

*Add $3$ to both sides of the equation:*
$$x^2 + 2x - 8 \geq 0$$

**Step 2:** To find the correct region of x-values, we first draw the corresponding graph. We need to find the x-intercepts of this graph by solving the corresponding quadratic equation by factorising the quadratic equation in the inequality.
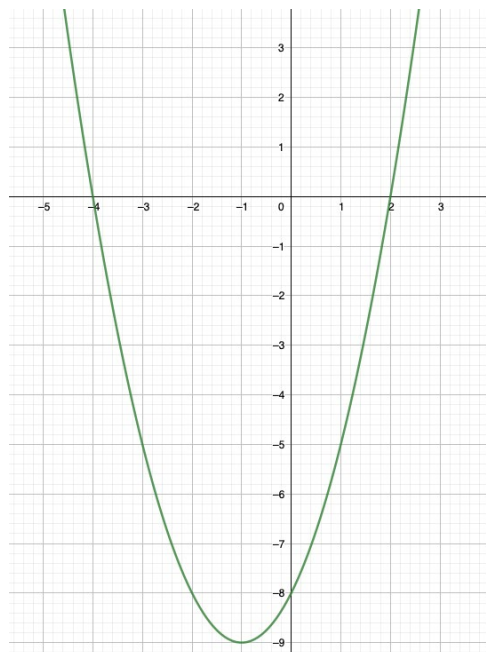
$$x^2 + 2x - 8 = 0$$
$$(x + 4)(x - 2) = 0$$
$$x + 4 = 0 \qquad or \qquad x - 2 = 0$$
$$x = -4 \qquad or \qquad x = 2$$

*The coordinates of the $x-$intercepts are $(-4,0)$ and $(2,0)$.*

**Step 3:** Using the coordinates of the $x-$intercept and the $y-$intercept, sketch a quadratic graph equivalent to the quadratic inequality.
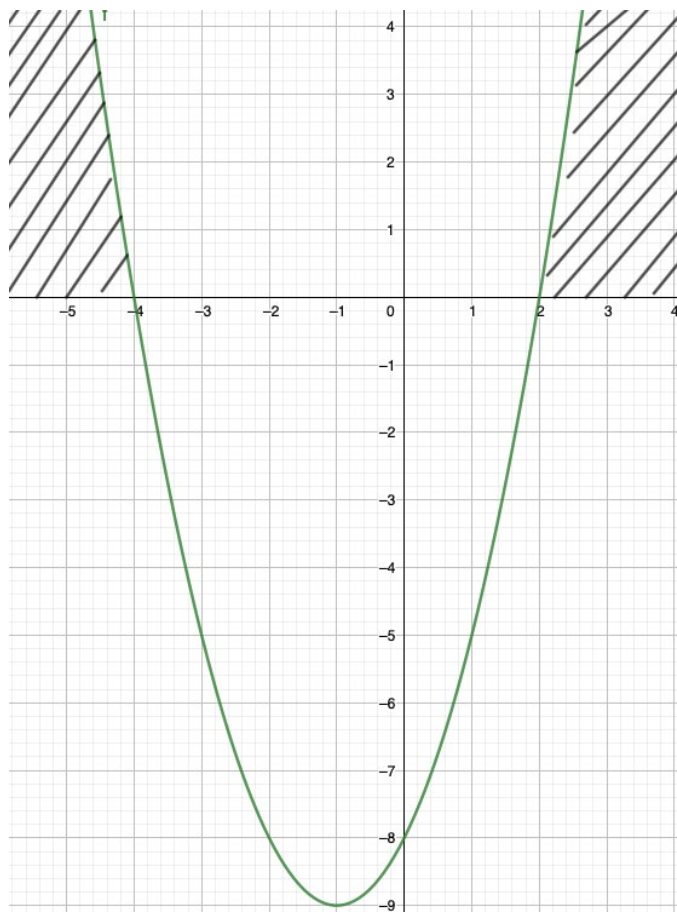
*$x-$intercepts: $(-4,0)$ and $(2,0)$*

*$y-$intercept: $(0,-8)$*

**Step 4:** Identify the required region which satisfies the quadratic inequality. Shade this region on the graph.

*For $x^2 + 2x - 8 \geq 0$ , the required region should be the area where the quadratic is greater than or equal to 0. In this case, the area should be the regions above the $x-$axis, as shaded in the graph below.*



**Step 5:** Write the answer of the required region in the form of an inequality. Since the shaded regions consist of two areas, the answer should be made up of two inequalities.

*The solution is $x \leq -4$ or $x \geq 2$.*

**Step 6:** Present the answer in a number line.
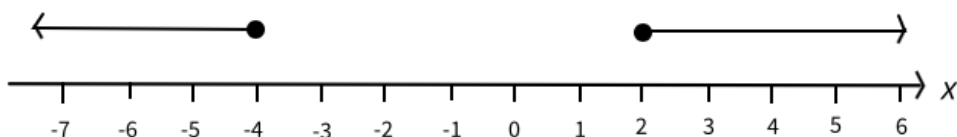


*In the number line, we use solid filled circles to denote the fact that $x = -4$ and $x = 2$ is included in the values for $x$.*

# Guided Example

**Solve the inequality $2x^2 - 4x - 4 \leq 6x + 8$.**

**Present your answer in a number line.**

**Step 1**: Rearrange the inequality so that the left-hand side of the inequality is in the form $ax^2 + bx + c$.

**Step 2:** To find the correct region of x-values, we first draw the corresponding graph. We need to find the x-intercepts of this graph by solving the corresponding quadratic equation by factorising the quadratic equation in the inequality.

**Step 3:** Using the coordinates of the $x-$intercept and the $y-$intercept, sketch a quadratic graph equivalent to the quadratic inequality

**Step 4:** Identify the required region which satisfies the quadratic inequality. Shade this region on the graph.

**Step 5:** Write the answer of the required region in the form of an inequality. Since the shaded regions consist of two areas, the answer should be made up of two inequalities.

**Step 6:** Present the answer in a number line.

1. Solve the following inequalities and present your answer in a number line.

a) $3x^2 + 2x < 14 + 2x^2 - 3x$

b) $x^2 + 5 > 30$

c)  $-2x^2 + 4 \geq x^2 + 9x - 8$

d)  $x^2 + 12 \geq -7x + 2$